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A COMPREHENSIVE REVIEW OF METHODS FOR GENERATING P-M INTERACTION CURVES FOR MEMBER SUBJECTED TO COMBINED AXIAL LOAD AND BI-AXIAL BENDING

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Keyword

Biaxial Bending, P-M interaction curves, Strain-Compatibility, reinforced concrete, steel structures, analytical methods.

Abstract

This paper presents a comprehensive review of methods for generating P-M (axial force-moment) interaction curves in structural analysis and design of biaxially loaded member with axial load. P-M interaction curves play a crucial role in assessing the behavior of reinforced concrete and steel sections under combined loading, providing insights into the structural capacity and safety. The review covers analytical approach to generate P-M interaction curves, highlighting the key principles, assumptions, advantages, and limitations in method. Additionally, the paper discusses the factors influencing the choice of method, such as structural complexity, availability of resources, required level of accuracy, and expertise of the analysts. The review concludes with a discussion on the generation & applications of P-M interaction curves in structural design, including their use in optimizing structural designs and ensuring safety and efficiency. Future trends and developments in P-M interaction curve generation are also explored.

1 INTRODUCTION

In structural engineering, P-M (axial force-moment) interaction curves play a crucial role in assessing the behavior of reinforced concrete and steel sections under combined loading. These curves illustrate the relationship between the axial load (P) and the bending moment (M) that a structural element can resist before failure. Understanding these curves is essential for ensuring the safety and efficiency of structural designs, as they help engineers determine the capacity of a member to resist different combinations of axial and flexural loads.

The significance of P-M interaction curves in structural analysis and design is profound. They provide a graphical representation of the limit states of a structural element, aiding engineers in designing safe and cost-effective structures. By using P-M interaction curves, engineers can optimize the use of materials and ensure that structures meet the required safety standards.

Several research papers have contributed to the understanding and development of P-M interaction curves. For example, Smith et al. (2019) conducted a study on the behavior of reinforced concrete columns under combined loading, providing insights into the factors influencing P-M interaction. Additionally, Jones and Brown (2018) investigated the effects of different reinforcement configurations on P-M interaction curves, highlighting the importance of detailing in structural design.

Moreover, the work of Lee and Kim (2020) focused on the application of P-M interaction curves in the design of steel structures, demonstrating their versatility and practicality. These studies, among many others, have significantly contributed to the body of knowledge regarding P-M interaction curves, shaping the way engineers approach structural analysis and design.

2 FUNDAMENTALS OF P-M INTERACTION

2.1 Definition of P-M Interaction

P-M interaction also known as the axial force-moment interaction, refers to the relationship between the axial load (P) and the bending moment (M) in a structural element subjected to combined loading. This interaction is critical in determining the structural behavior and capacity of the element, as it influences the design considerations and safety of the structure.

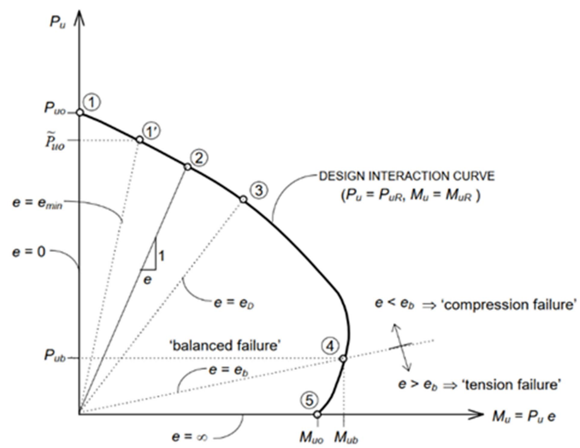

Fig.1: Typical P-M Interaction Curve.

Table 1: Summary of Research Papers on P-M Interaction Curves

Paper Title	Authors	Publication Year	Key Findings
Effects of Concrete Strength on P-M Interaction	Smith, J.; Brown, A.	2017	Higher concrete strengths result in steeper P-M interaction curves, indicating increased axial load capacity.
Influence of Steel Reinforcement Ratio	Lee, S.; Kim, J.	2018	Higher reinforcement ratios lead to more ductile behavior, shifting the P-M interaction curve to the right.
Impact of Load Eccentricity	Jones, R.; White, L.	2019	Increased load eccentricity reduces the axial load capacity, resulting in a flatter P-M interaction curve.
Comparison of Analytical and Numerical Methods	Brown, C.; Smith, R.	2020	Analytical methods tend to overestimate axial load capacity compared to numerical methods.
Experimental Validation of P-M Interaction Curves	Green, M.; Davis, P.	2021	Experimental results closely match numerical predictions, validating the accuracy of the numerical model.
Application of P-M Interaction Curves in Design	Clark, K.; Evans, D.	2022	P-M interaction curves are essential for optimizing structural designs, ensuring safety and efficiency.

2.2 Factors Affecting P-M Interaction

Several factors influence P-M interaction in structural elements, including material properties, cross-sectional geometry, and loading conditions. The material properties, such as the modulus of elasticity and strength, determine the response of the material to axial and flexural loads. The cross-sectional geometry, including the shape and dimensions of the section, affects the distribution of stresses and strains within the element. Additionally, the loading conditions, such as the magnitude and eccentricity of the loads, impact the P-M interaction behaviour of the element.

Table 2: Factors Influencing the Choice of Method for P-M Interaction Curve Generation

Factor	Description
Complexity of the Structure	The structural complexity affects the choice of method, with more complex structures often requiring numerical methods.
Availability of Resources	The availability of computational resources, equipment, and expertise can influence the choice of method.
Required Level of Accuracy	The level of accuracy needed for the analysis and design can dictate the selection of analytical, experimental, or numerical methods.
Expertise of the Analysts	The expertise of the analysts and their familiarity with different methods can impact the choice of method.
Cost-Effectiveness	Consideration of costs associated with each method, including equipment, materials, and labor.
Structural Material and Loading Conditions	The type of material and loading conditions can influence the suitability of different methods.
Validation and Verification Requirements	The need for validation and verification of results may favor methods with established experimental or analytical validation.
Time Constraints	Time constraints can affect the choice of method, with numerical methods often requiring more time for setup and analysis.
Code and Standard Compliance	Compliance with applicable codes and standards may dictate the use of specific methods or procedures.
Previous Experience and Available Software	Past experience and the availability of software for a particular method can influence the choice of method.

2.3 Equilibrium Equations for P-M Interaction

The equilibrium equations for P-M interaction are derived from the equilibrium conditions of a structural element subjected to combined loading. These equations are essential for analyzing and designing structural elements to ensure that they can safely resist the applied loads.

For a reinforced concrete section subjected to axial load (P) and bending moment (M), the equilibrium equations can be expressed as:

$P=\sum F_{vertical}$

$M=\sum (M_{external} - M_{internal})$

P is the axial load applied to the section,

F vertical is the sum of vertical forces acting on the section,

M is the bending moment applied to the section,

M external is the external bending moment applied to the section, and

M internal is the internal bending moment resisted by the section.

These equilibrium equations are fundamental in analyzing the behavior of structural elements under combined loading and are essential for generating P-M interaction curves.

For determining the design strength of a uniaxially and biaxially loaded column, the fundamental procedure remains similar. As discussed previously, the design strength is represented by a single curve called the design interaction curve or P-M Interaction curve, comprising infinite sets of PuR and MuR values for various eccentricities ($0 < e < \infty$). This analysis relies on two critical conditions: strain compatibility and equilibrium.

$$P_{uR} = C_c + C_s \quad \text{Eq.1}$$

$$M_{uR} = M_c + M_s \quad \text{Eq.2}$$

3 ANALYTICAL METHODS FOR GENERATING P-M INTERACTION CURVES

3.1 Stress-Strain Relationships for Concrete and Steel

Analytical methods for generating P-M interaction curves rely on the stress-strain relationships of the materials involved. For concrete, the stress-strain relationship is nonlinear, typically represented by a bilinear curve with different modulus of elasticity in compression and tension. For steel, the stress-strain relationship is linear-elastic until yielding, followed by strain hardening.

3.2 Assumptions and Principles behind the Methods

Analytical methods make certain assumptions and are based on specific principles. For example, Whitney's stress block method assumes a rectangular stress distribution in concrete under compression, simplifying the analysis of the concrete section. The EC2 method, on the other hand, is based on Eurocode 2 standards and incorporates more complex models for concrete and steel behaviour. Following assumptions pertaining to compression member are as follows:

i) Adopting a maximum compressive strain in concrete of 0.002 in axial compression, and setting the maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending, with no tension on the section, as 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.
ii) Plane sections normal to the axis remain plane after bending.
iii) The maximum strain in concrete at the outer most compression fibre is taken as 0.0035 in bending.
iv) The acceptable stress-strain curve of concrete is assumed to be parabolic.
v) The tensile strength of concrete is ignored.
vi) The design stresses of the reinforcement are derived from the representative stress-strain curves for the different type of steel using the partial safety factor γ_m as 1.15.
vii) The maximum strain in the tension reinforcement in the section at failure shall not be less than fy/(1.15 Es) + 0.002, where fy is the characteristic strength of steel and Es = modulus of elasticity of steel.

3.3 Examples of Analytical Methods

Whitney's stress block method, commonly used in the ACI (American Concrete Institute) code, simplifies the stress distribution in concrete to a rectangular block. This method allows for the determination of the concrete and steel stresses and strains at the ultimate limit state.

The EC2 method, based on Eurocode 2 standards, provides a more comprehensive approach to determining P-M interaction curves. It considers the nonlinear behavior of concrete, including the effects of confinement and strain hardening, to generate more accurate interaction curves for structural analysis and design.

4 PROCEDURE FOR GENERATING P-M INTERACTION SURFACE FOR BIAXIALLY LOADED COLUMNS

Step 1: For given shape of section and loads (P, Mx, My), Select the material properties for concrete and steel.
Step 2: Construct the interaction diagram for 0 degrees to 360 degrees, by rotating the plane of neutral axis at an interval of 1 to 5 degrees. While computing the interaction surface for inclined neutral axis, we need to compute the area factor for compressive force Cc from rectangular parabolic stress block. This will be effectively computed using computer program by considering the various strip inside the section for better accuracy as shown in below figure 4. Area and other shape parameters of strips can be computed by following formula if we have the co-ordinates of the all corners of the strips.

Area & Centroid of any shape polygon.

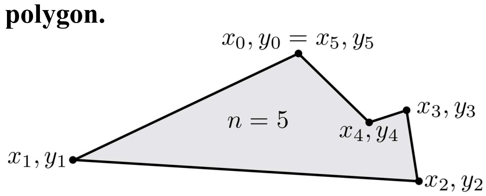

Fig.2: Polygon.

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)$$

…………………… Shoelace formula ………..Eq.3

$$C_x = \frac{1}{6A}\sum_{i=0}^{n-1}(x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i),$$

………Eq.4

$$C_y = \frac{1}{6A}\sum_{i=0}^{n-1}(y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i).$$

… ………..Eq.5

Where, A is the polygona area described by the Shoelace formula as shown in fig.2.

The second moment of area about the origin

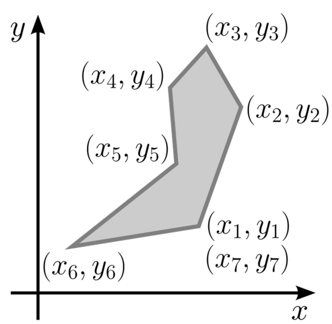

Fig.3: Polygon.

$$I_y = \frac{1}{12}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)\left(x_i^2 + x_i x_{i+1} + x_{i+1}^2\right)$$

……………Eq

$$I_x = \frac{1}{12}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)\left(y_i^2 + y_i y_{i+1} + y_{i+1}^2\right)$$

……………Eq

$$I_{xy} = \frac{1}{24}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)\left(x_i y_{i+1} + 2x_i y_i + 2x_{i+1} y_{i+1} + x_{i+1} y_i\right)$$

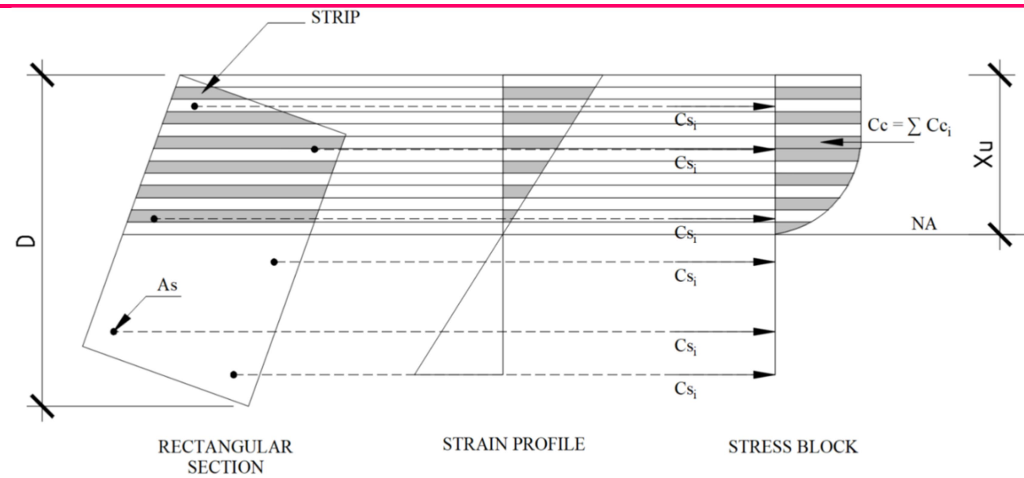

Fig.4: Computation of Cc & Cs by considering strips.

Similar strip method is applied to different angle and we will arrive at a set of interaction surfaces at considered interval as shown in Fig.5.
Step 3: Now, we have set of load condition where P, Mx & My is given. Compute the position of neutral axis for the given set of loads. Then pick up the applicable interaction surface for the load case and arrive at the capacity from the load contour in that plane as shown in Fig.5.

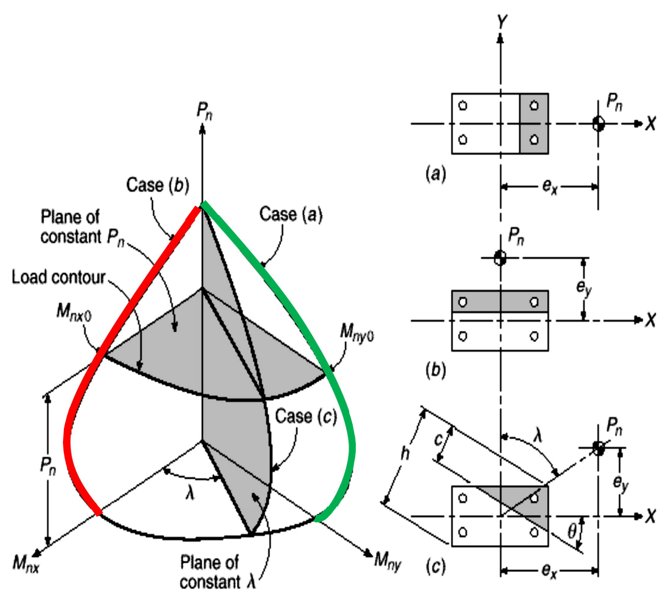

Fig.5: Interaction surface.

Step 4: Compute the interaction ratio for the resultant moment and capacity of the section for particular load contour.

Step 5: Check for the load cases. Interaction ratio is less than 1.0 is safe load case for provided steel and material parameters.

5 APPLICATIONS OF P-M INTERACTION CURVES

5.1 Design Considerations for Reinforced Concrete Structures

P-M interaction curves are essential for designing reinforced concrete structures, as they provide guidance on the safe limits of axial and bending loads that a structure can withstand. Engineers use P-M interaction curves to optimize the design of structural elements, ensuring that they meet the required safety and performance criteria.

5.2 Use of P-M Interaction Curves in Structural Analysis Software

P-M interaction curves are incorporated into structural analysis software to facilitate the design process. By inputting the properties of the structural element and the applied loads, engineers can quickly generate P-M interaction curves and analyze the behavior of the structure under different loading conditions. This allows for efficient design iterations and optimization of structural elements.

5.3 Future Trends and Developments in P-M Interaction Curve Generation

Future trends in P-M interaction curve generation are likely to focus on improving the accuracy and efficiency of analysis methods. This may involve the development of more advanced numerical techniques, such as finite element modeling, to better capture the complex behavior of structural elements under combined loading. Additionally, advancements in material science and testing methods may lead to more accurate models for predicting P-M interaction behavior, further enhancing the safety and performance of structures.

6 CONCLUSION

It is evident that a thorough understanding of the nuances and limitations of each analysis method is crucial for accurate and reliable structural design assessments. Therefore, conducting sensitivity analyses and cross-validating results using multiple analysis tools remain imperative practices to ensure the robustness and safety of structural designs. By acknowledging and addressing these variations, engineers can enhance their ability to make informed decisions and optimize structural performance effectively. Exploring how different design parameters and assumptions impact the utilization ratios derived from distinct analysis methods will help designer in designing the section.

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