

# INVESTIGATION ON MULTI-SPECTRAL IMAGES DENOISING BY INTRINSIC TENSOR SPARSITY REGULARIZATION

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## ABSTRACT

As a 3-order tensor, a multi-spectral image (MSI) has dozens of spectral bands, which can deliver more information for real scenes. However, real MSIs are often corrupted by noises in the sensing process, which will further deteriorate the performance of higher-level classification and recognition tasks. In this paper, we propose a Low-rank Tensor Dictionary Learning (LTDL) method for MSI denoising. Firstly, we extract blocks from the MSI and cluster them into groups. Then instead of using the exactly low-rank model, we consider a nearly lowrank approximation, which is closer to the latent lowrank structure of the clean groups of real MSIs. In addition, we propose to learn an spatial dictionary and an spectral dictionary, which contain the spatial features and spectral features respectively of the whole MSI and are shared among different groups. Hence the LTDL method utilizes both the latent low-rank prior of each group and the correlation of different groups via the shared dictionaries. Experiments on synthetic data validate the effectiveness of dictionary learning by the LTDL. In specific, we construct a new tensor sparsity measure, called intrinsic tensor sparsity (ITS) measure, which encodes both sparsity insights delivered by the most typical Tucker and CANDECOMP/ PARAFAC (CP) low-rank decomposition for a general tensor. Then we build a new MSI denoising model by applying the proposed ITS measure on tensors formed by non-local similar patches within the MSI. The intrinsic GCS and NSS knowledge can then be efficiently explored under the regularization of this tensor sparsity measure to finely rectify the recovery of a MSI from its corruption. A series of experiments on simulated and real MSI denoising problems show that our method outperforms all state-of-the-arts under comprehensive quantitative performance measures.

## 1. INTRODUCTION

A multi-spectral image (MSI) has dozens of spectral bands, where the wavelengths may range from infrared to ultra-violet. Compared with a RGB image which only has three spectral bands, an MSI provides more information which reveals features of the object hidden in the spectral domain. However, in many cases, MSIs suffer from corruptions or noises in the sensing process [2]. As a low level image processing technique, MSI denoising is key to many high-level computer vision tasks, such as segmentation and classification whose performance highly relies on the quality of the data.

As a model driven approach, dictionary learning methods have been used to find the basic atoms which comprise various signals of a training

dataset. By using a learned dictionary of some signal ensembles, noises can be effectively removed via solving a sparse signal recovery problem for each patch of an image. For MSI denoising, applying the traditional dictionary learning methods, e.g., K-SVD [1], for each band leads to the poor performance, as it fails to exploit spectral information in MSIs.

Tensor dictionary learning, which keeps the multidimensional structure of tensors, has attracted growing interests of researchers to process images in the past years. Based on CANDECOMP/PARAFAC (CP) decomposition, Duan et al. extend the K-SVD method for tensors, where a higher order tensor dictionary is learned and each atom of the dictionary is a rank-one tensor. By using the Tucker model of tensors, Zubair and Wang

propose to learn multiple orthogonal dictionaries along different modes of tensors, where the core tensor have sparse nonzero elements. In Qi et al. divide an MSI into small 3-order tensor blocks, and learn overcomplete dictionaries for each mode of the blocks via a two-phase block-coordinate-relaxation approach that includes sparse coding and dictionary updating. However, they fails to further employ all the structural information embedded in images.

In this paper, we propose a novel tensor dictionary learning model for the task of MSI denoising by combinationally considering two characteristics of MSI into a single framework: nonlocal similarity in space and global correlation in spectrum. On one hand, a typical natural scene contains a collection of similar local patches all over the space, composing of homologous aggregation of microstructures. By averaging among these nonlocally similar patches, the spatial noise is expected to be prominently alleviated. On the other hand, an MSI contains a large amount of spectral redundancy. That is, images obtained over different bands are always highly correlated. Through extracting the major components from these globally correlated spectrum information, the spectral MSI noise (the minor components) is expected to be eliminated. Both characteristics

can be easily understood by seeing Fig. 1. In our model, we employ a grouped sparsity regularizer to impose similar MSI patches to share the same dictionary atoms in their sparse decomposition to implicitly noise among these patches. Furthermore, by assuming redundant dictionaries over both the space and spectrum, the proposed tensor dictionary learning model can be readily decomposed into a series of low-rank tensor approximation problems. Each of these problems corresponds to a spectral dimensionality reduction model conducted by the spectral correlation property of MSIs, and can be easily solved by some off-the-shelf higher order statistics. The spectral redundancy problem can thus be alleviated.

## 2. RELATEDWORK

To decompose images with noises, one require the priors of the different signals. The “No Free Lunch” theory in machine learning suggests that all algorithms perform the same for the randomised data and we can achieve good performance only when the data has some structure and some appropriate model is used. In the past decade, various model driven methods have been proposed for image denoising, and some of them are summarized

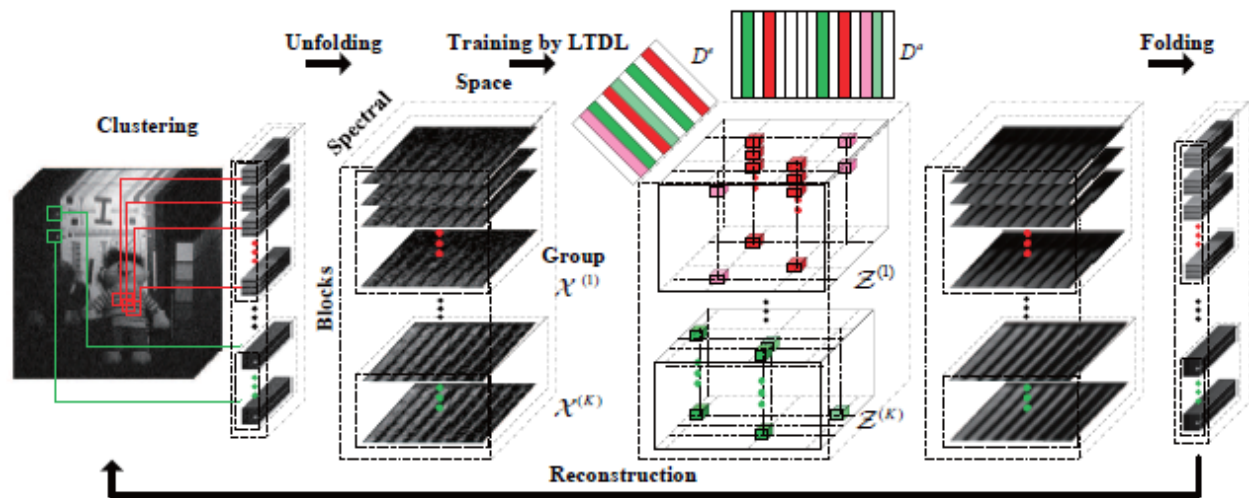


Figure 1: Flowchart of the proposed MSI denoising method.

The first category of image denoising methods uses different filters, e.g., the mean-value filter and the Wiener filter, to exploit the local correlation between adjacent pixels and/or the non-local correlation between similar small patches/blocks of an image. Another category of widely used methods considers the property that natural images or clustered groups of their patches usually exhibit low-rank structures. Different low-rank approximation methods have been proposed for image denoising such as nuclear norm regularization, Tucker low-rank decomposition and CP low-rank decomposition. Based on the sparsity model, dictionary learning methods assume that image patches/blocks are linear compositions of very few atoms selected from a dictionary. Dictionary learning and sparse representation is firstly applied for 2D image denoising, and then extended to higher-order image denoising such as TenSR. In recent years, researchers consider to exploit both the sparsity model and low rank model to better utilize the prior of the groups extracted from an MSI. Unfortunately, in these recent methods, dictionaries (or called factors) are learned separately for each tensor group, which deviates from the principle of dictionary learning and significantly increases the total number of dictionary atoms. Our work differs with the existing methods for MSI denoising. Instead of using the exact low-rank model, we consider a nearly low-rank structure for each tensor group of an MSI, and learning dictionaries that are shared among all groups.

### 3. DECOMPOSABLE NONLOCAL MSI DICTIONARY LEARNING

In this section, we first introduce the tensor dictionary learning (DL) model, and then present the main idea of our decomposable nonlocal MSI DL model and the related algorithm. The parameter setting problems are also discussed thereafter.

#### 3.1. From Image DL to MSI DL

We first briefly introduce the traditional DL model for image restoration. For a set of image patches (ordered lexicographically as column vectors)  $\{\mathbf{x}_i\}_{i=1}^n \subset \mathbb{R}^d$ , where  $d$  is the dimensionality and  $n$  is the number of image

patches, DL aims to calculate the dictionary  $\mathbf{D} = [\mathbf{d}_1, \dots, \mathbf{d}_m] \in \mathbb{R}^{d \times m}$ , composed by a collection of atoms  $\mathbf{d}_i$  ( $m > d$ , implying that the dictionary is redundant), and the coefficient matrix  $\mathbf{Z} = [\mathbf{z}_1, \dots, \mathbf{z}_n] \in \mathbb{R}^{m \times n}$ , composed by the representation coefficients  $\mathbf{z}_i$  of  $\mathbf{x}_i$ , by the following optimization model [1]:

$$\min_{\mathbf{D}, \mathbf{z}_1, \dots, \mathbf{z}_n} \sum_{i=1}^n \|\mathbf{x}_i - \mathbf{D}\mathbf{z}_i\| \quad \text{s.t.} \quad \mathcal{P}(\mathbf{z}_i) \leq k, \quad (1)$$

The proposed LTDL method for MSI denoising In this subsection, we introduce the proposed MSI denoising method that learns shared tensor overcomplete dictionaries and considers the nearly low-rank structure. Two dictionaries, i.e.,  $\mathbf{D}_a$  and  $\mathbf{D}_e$ , are learned from all tensor groups of an MSI, where  $\mathbf{D}_a$  corresponds to the spatial domain and  $\mathbf{D}_e$  corresponds to the spectral domain. Corresponding to the spatial dictionary and the spectral dictionary, respectively. The proposed LTDL method for MSI denoising is formulated as follows:

$$\begin{aligned} \min_{\substack{\mathbf{D}^a, \mathbf{D}^e \\ \{\mathbf{Z}^{(k)}, \mathcal{G}^{(k)}\}_{U_i^{(k)}}}} \sum_{k=1}^K \left( \left\| \mathcal{X}^{(k)} - \mathbf{Z}^{(k)} \times_1 \mathbf{D}^a \times_2 \mathbf{D}^e \right\|_F^2 + \lambda_s \left\| \mathbf{Z}^{(k)} \right\|_1 \right. \\ \left. + \lambda_r \left\| \mathbf{Z}^{(k)} \times_1 \mathbf{D}^a \times_2 \mathbf{D}^e - \mathcal{G}^{(k)} \times_1 U_1^{(k)} \times_2 U_2^{(k)} \times_3 U_3^{(k)} \right\|_F^2 \right) \\ \text{s.t.} \quad \left\| \mathbf{D}^a(:, r) \right\|_2^2 = 1 \text{ for } r = 1, \dots, \tau_a d_L d_W \\ \left\| \mathbf{D}^e(:, r) \right\|_2^2 = 1 \text{ for } r = 1, \dots, \tau_e H, \end{aligned}$$

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**Algorithm 1** Algorithm for MSI denoising

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**Input:** the MSI of  $\mathcal{H} \in \mathbb{R}^{L \times W \times H}$

**Output:** denoised MSI  $\mathcal{H}_{de}$ , spatial dictionary  $D^a$  and spectral dictionary  $D^e$

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1: Construct groups  $\mathcal{X}^{(k)} (k = 1, \dots, K)$  by the extracting, clustering and unfolding process of the  $\mathcal{H}$ 
2: Initialize  $D^a, D^e, \{\mathcal{C}^{(k)}, \mathcal{Z}^{(k)}, \mathcal{Y}^{(k)}\}_{k=1}^K$ 
3: while not converged do
4:   for  $k = 1 : K$  do
5:     update  $\mathcal{T}^{(k)}$  by (9)
6:     update  $\mathcal{Z}^{(k)}$  via folding  $\mathcal{Z}_{(3)}^{(k)}$  by (11)
7:     update  $\mathcal{C}^{(k)}$  by (13)
8:   end for
9:   update  $D^a$  by (16)
10:  update  $D^e$  by (18)
11:  for  $k = 1 : K$  do
12:    update  $\mathcal{Y}^{(k)}$  by (19)
13:  end for
14:   $\rho := \mu\rho$ 
15: end while
16: Reconstruct groups  $\hat{\mathcal{X}}^{(k)} = \mathcal{Z}^{(k)} \times_1 D^a \times_2 D^e (k = 1, \dots, K)$ 
17: Aggregate  $\hat{\mathcal{X}}^{(k)}$  to form the denoised MSI  $\mathcal{H}_{de}$ 

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### Intrinsic tensor sparsity measure

We first briefly review two particular forms of tensor

decomposition, both containing insightful understanding of tensor sparsity: Tucker decomposition [27] and CP decomposition [14].

$$\mathcal{X} = \mathcal{S} \times_1 U_1 \times_2 U_2 \times_3 \dots \times_N U_N,$$

Tucker decomposition considers the low-rank property of the vector subspace unfolded along each of its modes. Such a sparsity understanding naturally conducts a block shape for the coefficients affiliated from all combinations of tensor subspace bases, represented by the core tensor term. It, however, has not considered the fine-grained sparsity configurations inside this coefficient tensor. Specifically, if we assume that the subspace bases along each mode have been sorted based on their importance for tensor

representation, then the value of elements of the core tensor will show an approximate descending order along each of tensor modes. Along some modes, the corresponding tensor factor might have strong correlations across data, and then the coefficients in the core tensor along this mode tends to be decreasing very fast to zeroes.

While for those modes with comparatively weaker correlation, albeit still approximately decreasing along the mode, the core tensor elements might be all non-zeroes. Fig. 2 depicts visualization for facilitating the understanding of the above analysis.

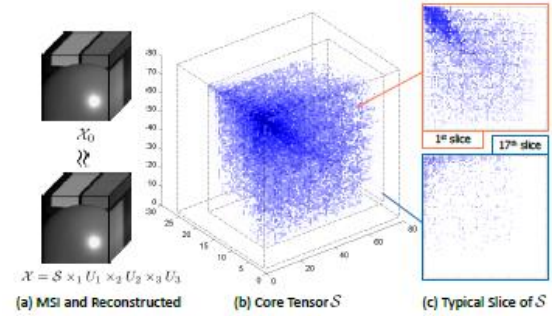


Figure 2. (a) An MSI  $\mathcal{X}_0$  2 R80\_80\_26 (upper) and a nearly perfect reconstruction  $\mathcal{X}_0$  (PSNR=61.25). (b) Core tensor  $\mathcal{S}$  2R69. Note that 78:4% of its elements are zeroes and more than half of them are very small. (c) Typical Slices of  $\mathcal{S}$ , where deeper color of the element represents a larger value of it.

### CONCLUSION

This paper presents an effective tensor dictionary learning method for restore high dimensional MSIs. The proposed LTDL method exploits the nearly low-rank structure in a group of similar blocks in the natural MSI and also exploits shared dictionaries among different groups, which makes the proposed method distinct to existing methods.

Experimental results show the superior performance of the proposed method for denoising MSIs with both simulated corruptions and real corruptions. We have also designed an efficient ADMM algorithm to solve the model. The experiments on simulated and real MSI denoising have substantiated the superiority of the proposed method beyond state-of-the-arts.

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